UDC 532.5

ON THE PROBLEM OF FLOW OF A HEAVY FLUID WITH TWO FREE SURFACES

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The problem of steady plane flow of a perfect ponderable fluid bounded by solid polygonal sections and two free sutfaces of finite length is considered in the exact nonlinear formulation. It is shown that the problem is solvable for sufficiently high Froude numbers. The solvability of a particular problem of this kind was proved earlier in [1] on certain simplifying assumptions, while solvability of the problem of flow of a ponderable fluid with a single free surface was investigated in [2 – 5].

1. The steady potential flow of an incompressible ponderable fluid whose boundaries consist of solid polygonal sections and free surfaces AB and CD of finite length is considered in the plane $z = x + iy$. Isolated hydrodynamic singularities may be present in the flow. One of the possible patterns of fluid motion is shown in Fig. 1.

Let in the plane of the auxiliary variable $\zeta = \xi + i\eta$ the rectangle with vertices $0, \pi / 2, \pi / 2 + \pi\tau / 2$ and $\pi\tau / 2$ $(\tau = i|\tau|)$ be the conformal image of the flow region with its free surfaces represented by the horizontal sides of that rectangle (Fig. 2). We denote the inner region of the rectangle by D.

The derivative of the complex potential $dw / d\zeta$ in the ζ-plane is readily constructed at zeros and poles [6] (their number and multiplicity are determined by the flow pattern and their position in the closed region $\bar{D}$ is assumed to be known). Function $dw / d\zeta$ is elliptic of periods π and $\pi\tau$, and is of the form

$$\frac{dw}{d\zeta} = |\varphi_0| F(\zeta) = \varphi_0 \theta_1(2\zeta) \prod_m [\theta_1(\zeta - ia_m)\theta_1(\zeta + ia_m)]^{d_m} \times \tag{1.1}$$

$$\prod_n [\theta_2(\zeta - ib_n)\theta_2(\zeta + ib_n)]^{c_n} \times$$

$$\prod_k [\theta_1(\zeta - \zeta_k)\theta_1(\zeta + \zeta_k)\theta_1(\zeta - \bar{\zeta}_k)\theta_1(\zeta + \bar{\zeta}_k)]^{\varkappa_k}$$

$$\Big(2 + \sum_m d_m + \sum_n c_n + 2\sum_k \varkappa_k = 0, \ a_m, b_n \in (0, \pi|\tau| / 2), \ \zeta_k \in D\Big)$$

where φ_0 is a constant whose dimension is that of the velocity potential, $\theta_1(\zeta)$ and $\theta_2(\zeta)$ are theta functions, d_m, c_n and $\varkappa_k$ are integers, and ζ_0 is the image of an infinitely distant point of plane z, provided it exists and lies in D.

Let us introduce in the analysis the Joukowski function

$$\chi(\zeta) = \ln\left(V_0 \frac{dz}{dw}\right) = r + i\theta, \quad r = \ln\frac{V_0}{V} \tag{1.2}$$

where V is the absolute velocity, V_0 is its value at point A, and θ is the angle of inclination of velocity to the x-axis.

Assuming that images of the polygon vertices are specified in the ζ-plane and that the angles of inclination of polygon sections and the position and the kind of singular

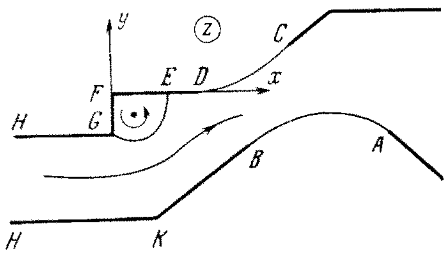

Fig. 1

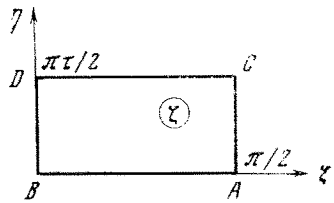

Fig. 2

points of function $\chi(\zeta)$ are known, we construct the piecewise constant functions $a(\eta)$ and $b(\eta)$

$$a(\eta) = \theta(i\eta), \quad b(\eta) = \theta(\pi / 2 + i\eta) \qquad (1.3)$$

We ascribe to function $\theta(\zeta)$ the values it assumes while continuously changing with the motion of point ζ along the contour $DBAC$ and bypassing singular points along arcs of infinitely small circles. In accordance with the accepted rule for the scheme represented in Fig. 1, we have: along DE $\theta = 0$, along EF $\theta = \pi$, along FG $\theta = 3\pi / 2$, along GH and HK $\theta = 2\pi$ and so on.

Let us represent $\chi(\zeta)$ in the form of the sum

$$\chi(\zeta) = \chi_0(\zeta) + f^*(\zeta) \qquad (1.4)$$

where $\chi_0(\zeta)$ is the Joukowski function in the case of a weightless fluid flow according to the considered pattern, and $f^*(\zeta)$ is a function which is analytic in D and continuous in $\overline{D}$. Boundary conditions for $\chi_0(\zeta)$ are of the form

$$\begin{aligned} &\mathrm{Re}\chi_0(\xi) = 0 \quad (0 \leqslant \xi \leqslant \pi / 2) \\ &\mathrm{Im}\chi_0(i\eta) = a(\eta) \quad (0 \leqslant \eta \leqslant \pi|\tau| / 2) \\ &\mathrm{Im}\,\chi_0(\pi / 2 + i\eta) = b(\eta) \quad (0 \leqslant \eta \leqslant \pi|\tau| / 2) \\ &\mathrm{Re}\,\chi_0(\pi\tau / 2 + \xi) = r_1 \quad (0 \leqslant \xi \leqslant \pi / 2) \end{aligned} \qquad (1.5)$$

where r_1 is a constant which remains to be determined.

We denote the points of discontinuity of functions $a(\eta)$ and $b(\eta)$ by α_j and β_l, respectively. It is readily seen that the derivative $d\chi_0 / d\zeta$ at points $\zeta = i\alpha_j, \pi/2 + i\beta_l$, and ζ_k $(k \neq 0)$ has first order poles with residues expressed, respectively, by

$$\delta_j = [a(\alpha_j + 0) - a(\alpha_j - 0)]\pi^{-1}, \quad \nu_l = [b(\beta_l - 0) - b(\beta_l + 0)]\pi^{-1}$$

and $-\varkappa_k$. The derivative $d\chi_0 / d\zeta$ has no other singularities in $\overline{D}$.

Taking into consideration that along the sides of rectangle $ABDC$ $\mathrm{Re}\,(d\chi_0 / d\zeta) = 0$ and using the principle of symmetry, we extend $d\chi_0 / d\zeta$ over the whole ζ-plane. As the result we obtain an elliptic function of periods π and $\pi\tau$, which we represent in the form of a linear combination of logarithmic derivatives of theta functions (see [7] p. 350). Subsequent integration yields for $\chi_0(\zeta)$

$$\chi_0(\zeta)=\ln\left\{\prod_j\left[\frac{\theta_1(\zeta-i\alpha_j)}{\theta_1(\zeta+i\alpha_j)}\right]^{\delta_j}\prod_l\left[\frac{\theta_2(\zeta-i\beta_l)}{\theta_2(\zeta+i\beta_l)}\right]^{\nu_l}\right\}-$$
$$\ln\prod_k^{k\neq 0}\left[\frac{\theta_1(\zeta-\zeta_k)\,\theta_1(\zeta+\bar\zeta_k)}{\theta_1(\zeta-\bar\zeta_k)\,\theta_1(\zeta+\zeta_k)}\right]^{\varkappa_k}+iA\zeta+iB \tag{1.6}$$

Real constants A and B are determined by the conditions

$$\operatorname{Im}\chi_0(0)=a(0),\qquad \operatorname{Im}\chi_0(\pi/2)=b(0)$$

which reduce to the form

$$-\pi\sum_j\delta_j+B=a(0) \tag{1.7}$$
$$\pi\sum_l\nu_l-2\pi\sum_k^{k\neq 0}\varkappa_k+\frac{1}{2}\pi A+B=b(0)$$

It can be readily ascertained that, when conditions (1.7) are satisfied, function $\chi_0(\zeta)$ defined by formula (1.6) has in $\bar{D}$ the required singularities and satisfies boundary conditions (1.5), and

$$r_1=-2\sum_j\alpha_j\delta_j-2\sum_l\beta_l\nu_l+4\sum_k^{k\neq 0}\varkappa_k\operatorname{Im}\zeta_k-\frac{1}{2}A\pi|\tau|$$

2. With the y-axis directed vertically upward it is possible to reduce by differentiation the conditions of pressure constancy at each free surface to the form

$$e^{-3r}\frac{dr}{d\xi}=\frac{g}{V_0^3}\frac{d\varphi}{d\xi}\sin\theta \tag{2.1}$$

where g is the acceleration of gravity and φ is the velocity potential.

Taking into account equalities (1.1) – (1.5) and (2.1), we obtain for function $f^*(\zeta)$ the following boundary value problem:

$$\exp(-3\lambda_k^*)\,d\lambda_k^*/d\xi=\gamma\rho_k\sin(T_k+\mu_k^*)\quad(k=0,1;\ 0\leqslant\xi\leqslant\pi|\tau|/2) \tag{2.2}$$

$$\operatorname{Re}f^*(\pi/2)=0 \tag{2.3}$$

$$\operatorname{Im}f^*(i\eta)=\operatorname{Im}f^*(\pi/2+i\eta)=0\quad(0\leqslant\eta\leqslant\pi|\tau|/2) \tag{2.4}$$

where

$$\gamma=g|\varphi_0|/V_0^3$$
$$T_k=\operatorname{Im}\chi_0(k\pi\tau/2+\xi),\qquad \rho_k=e^{3kr_1}F(k\pi\tau/2+\xi) \tag{2.5}$$
$$\lambda_k^*=\operatorname{Re}f^*(k\pi\tau/2+\xi),\quad \mu_k^*=\operatorname{Im}f^*(k\pi\tau/2+\xi)$$

Denoting

$$3\operatorname{Re}f^*(\pi\tau/2+\pi/2)=\mu_2^* \tag{2.6}$$

from conditions (2.2) and (2.3) we obtain

$$d\lambda_k^*/d\xi=\gamma\rho_k\exp(k\mu_2^*)\sin(T_k+\mu_k^*)\times \tag{2.7}$$

$$\left[1 + 3\gamma \exp(k\mu_2^*) \int_{\xi}^{\pi/2} \rho_k \sin(T_k + \mu_k^*)\, d\xi\right]^{-1} \quad (k = 0.1)$$

Let $h(\xi)$, $h_0(\xi)$ and $h_1(\xi)$ be real functions continuous along segment $[0, \pi / 2]$. We introduce operator K

$$K(h_0, h_1) = \frac{i}{\pi} \int_0^{\pi/2} h_0(t) \ln \frac{\theta_1(\zeta - t)}{\theta_1(\zeta + t)}\, dt - \frac{i}{\pi} \int_0^{\pi/2} h_1(t) \ln \frac{\theta_4(\zeta - t)}{\theta_4(\zeta + t)}\, dt \qquad (2.8)$$

It can be readily ascertained that function

$$f(\zeta) = K(h_0, h_1) \qquad (2.9)$$

is regular in D, continuous in $\bar{D}$ and satisfies conditions

$$\operatorname{Im} f(i\eta) = \operatorname{Im} f(\pi / 2 + i\eta) = 0 \quad (0 \leqslant \eta \leqslant \pi\, |\tau| / 2) \qquad (2.10)$$

$$\frac{d}{d\xi} \operatorname{Re} f\left(\frac{k\pi\tau}{2} + \xi\right) = h_k(\xi) \quad \left(k = 0, 1;\ 0 \leqslant \xi \leqslant \frac{\pi}{2}\right)$$

$$\operatorname{Re} f(\pi / 2) = 0$$

Introducing notation

$$\lambda_k = \operatorname{Re} f(k\pi\tau / 2 + \xi), \quad \mu_k = \operatorname{Im} f(k\pi\tau / 2 + \xi) \qquad (2.11)$$

$$\lambda_k' = d\lambda_k / d\xi \ (k = 0, 1), \quad \mu_2 = 3 \operatorname{Re} f(\pi\tau / 2 + \pi / 2)$$

and operators

$$D_0(h) = \frac{1}{2\pi} \int_0^{\pi/2} h(t) \ln\left[\frac{\theta_1(\xi - t)}{\theta_1(\xi + t)}\right]^2 dt$$

$$D_1(h) = \frac{1}{2\pi} \int_0^{\pi/2} h(t) \ln\left[\frac{\theta_4(\xi - t)}{\theta_4(\xi + t)}\right]^2 dt$$

$$D_2(h) = \frac{6}{\pi} \int_0^{\pi/2} h(t)\, t\, dt$$

from (2.8) and (2.9) we obtain relationships

$$\mu_0 = D_0(\lambda_0') - D_1(\lambda_1'), \quad \mu_1 = D_1(\lambda_0') - D_0(\lambda_1') \qquad (2.12)$$

$$\mu_2 = D_2(\lambda_1') - D_2(\lambda_0')$$

Let d be a real number. We introduce operators P_0 and P_1

$$P_k(h, d) = \gamma \rho_k \exp(kd) \sin(T_k + h) \times$$

$$\left[1 + 3\gamma \exp(kd) \int_{\xi}^{\pi/2} \rho_k \sin(T_k + h)\, d\xi\right]^{-1} \quad (k = 0, 1)$$

Allowing for (2.7) for the determination of functions μ_0^*, μ_1^*, and constant μ_2^* from (2.12) we obtain the following system of operator equations:

$$\mu_0 = D_0(P_0(\mu_0, \mu_2)) - D_1(P_1(\mu_1, \mu_2)) \qquad (2.13)$$

$$\mu_1 = D_1(P_0(\mu_0, \mu_2)) - D_0(P_1(\mu_1, \mu_2))$$

$$\mu_2 = D_2(P_1(\mu_1, \mu_2)) - D_2(P_0(\mu_1, \mu_2))$$

Let C be the space of functions continuous in the interval $[0, \pi / 2]$ and E the space of real numbers. Introducing the Banach space

$$B = C \times C \times E = \{\nu = (\mu_0, \mu_1, \mu_2): \mu_0, \mu_1 \in C, \mu_2 \in E\}$$

with norm

$$\|\nu\|_B = \|\mu_0\|_C + \|\mu_1\|_C + \|\mu_2\|_E$$

and operator A defined over B by the equalities

$$\begin{aligned} A(\nu) &= A_0(\nu) \times A_1(\nu) \times A_2(\nu) \\ A_0(\nu) &= D_0(P_0(\mu_0, \mu_2)) - D_1(P_1(\mu_1, \mu_2)) \\ A_1(\nu) &= D_1(P_0(\mu_0, \mu_2)) - D_0(P_1(\mu_1, \mu_2)) \\ A_2(\nu) &= D_2(P_1(\mu_1, \mu_2)) - D_2(P_0(\mu_0, \mu_2)) \end{aligned} \tag{2.14}$$

we represent system (2.13) in the form of the operator equation

$$\nu = A(\nu) \tag{2.15}$$

3. Let us investigate the properties of the introduced operators.

Lemma. Operators D_0 and D_1 transform space C into itself.

Let $h(\xi) \in C$ and $\tau_0(\xi) = D_0(h(\xi))$. Let us consider the remainder $\tau_0(\xi) - \tau_0(\xi')$. Using the expansion of function $\theta_1(\xi)$ into an infinite product (see [7] p. 344), the equalities [8]

$$\ln(1 - q^{2n} e^{2i\xi}) = -\sum_{k=1}^{\infty} \frac{1}{k} q^{2nk} e^{2ik\xi} \quad (q = e^{i\pi\tau})$$

$$\ln\left|\frac{\sin(\xi - t)}{\sin(\xi + t)}\right| = -2\sum_{k=1}^{\infty} \frac{1}{k} \sin 2k\xi \sin 2kt$$

the Cauchy-Buniakowski inequality, and the Parceval equality, we obtain

$$|\tau_0(\xi) - \tau_0(\xi')| \leqslant \frac{2}{\pi}\|h\|_c \left\{\sum_{n=0}^{\infty}\sum_{k=1}^{\infty} \frac{1}{k^2} q^{4nk} |\sin 2k\xi - \sin 2k\xi'|^2\right\}^{1/2} \tag{3.1}$$

Since the series in the right-hand side of expression (3.1) is uniformly convergent and for $\xi \to \xi'$ tends to zero, hence $\tau_0(\xi) \in C$. The validity of this lemma with respect to operator D_1 is proved in a similar manner. The norms of operators D_0 and D_1 are determined as in [1, 9] by

$$\|D_0\|_C = d_0 = \frac{2}{\pi}\left\{G + \sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\left(\frac{2q^{2n}}{1+q^{4n}}\right)^{2k-1} \frac{(2k-2)!!}{(2k-1)(2k-1)!!}\right\} \tag{3.2}$$

$$\|D_1\|_c = d_1 = \frac{2}{\pi}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\left(\frac{2q^{2n-1}}{1+q^{4n-2}}\right)^{2k-1} \frac{(2k-2)!!}{(2k-1)(2k-1)!!}$$

where G is the Catalan constant. Derivation of the norm of operator D_2 is elementary

$$\|D_2\|_E = 3\pi / 4 \tag{3.3}$$

According to (1.1) and (1.6) ρ_k and $T_k \in C$ $(k = 0, 1)$ (note that when the free surfaces contain infinitely distant points, the form of function $dW / d\zeta$ is different from

(1.1) and $\rho_k \notin C$).

If μ_0 and $\mu_1 \in C$, and $\mu_2 \in [0, r_2]$, then for

$$\gamma < \omega^{-1}e^{-r_2}, \quad \omega = 3\pi \max_{k=0,1} \|\rho_k\|_C \tag{3.4}$$

the relationships

$$P_k(\mu_k, \mu_2) \in C \ (k = 0,1) \tag{3.5}$$
$$\|P_k(\mu_k, \mu_2)\|_C \leqslant \gamma\omega e^{r_2}(1 - \gamma\omega e^{r_2})^{-1}(3\pi)^{-1}$$
$$\|A_2(\nu)\|_E \leqslant {}^1/_2\, \gamma\omega e^{r_2}(1 - \gamma\omega e^{r_2})^{-1}$$

are valid. For

$$\gamma < 2r_2[\omega(1 + 2r_2)e^{r_2}]^{-1} \tag{3.6}$$

condition (3.4) is satisfied, and (3.5) yields the estimate $\|A_2(\nu)\|_E < r_2$. This, with the lemma taken into account, implies that operator A transforms the closed set $B_2 = C \times C \times [0, r_2]$ ($B_2 \subset B$) into itself.

Let $\nu' = (\mu_0', \mu_1', \mu_2') \in B_2$, $\nu'' = (\mu_0'', \mu_1'', \mu_2'') \in B_2$, and γ satisfies condition (3.4). Then, after some transformations, for $k = 0,1$ we obtain

$$\|P_k(\mu_k', \mu_2') - P_k(\mu_k'', \mu_2'')\|_C \leqslant \gamma\omega e^{r_2}[k\, \|\mu_2' - \mu_2''\|_E + (1 + 2\gamma\omega e^{r_2})\, \|\mu_k' - \mu_k''\|_C]\, (1 - \gamma\omega e^{r_2})^{-2}(3\pi)^{-1} \tag{3.7}$$

Allowing for (3.2), (3.3) and (3.7), from (2.14) we obtain

$$\|A(\nu') - A(\nu'')\|_B = \|A_0(\nu') - A_0(\nu'')\|_C + \|A_1(\nu') - A_1(\nu'')\|_C + \|A_2(\nu') - A_2(\nu'')\|_E \leqslant \alpha\, \|\nu' - \nu''\|_B$$
$$\alpha = \alpha(r) = \gamma\omega e^{r_2}(1 + 2\gamma\omega e^{r_2})(1 - \gamma\omega e^{r_2})^{-2}\, b^{-1}$$
$$b = 3\pi(3\pi / 4 + d_0 + d_1)^{-1}$$

It can be readily shown that $\alpha < 1$, when

$$\gamma < \beta\omega^{-1}e^{-r_2}, \quad \beta = (2b + 1 - \sqrt{1 + 12b})\,(2b - 4)^{-1} \tag{3.8}$$

Since $\beta < 1$, hence for

$$\gamma < \varphi(r_2) = 2\beta r_2[\omega(1 + 2r_2)e^{r_2}]^{-1}$$

the inequalities (3.4), (3.6) and (3.8) are simultaneously satisfied, and operator A satisfies the conditions of applicability of the method of contractive mappings.

The curve $\gamma = \varphi(r_2)$ ($0 \leqslant r_2 < \infty$) shown in Fig. 3 passes through the coordinate origin and asymptotically tends to the axis of abscissas when $r_2 \to \infty$. For $r_2 = {}^1/_2$ function $\varphi(r_2)$ reaches its single maximum $\varphi({}^1/_2) = ({}^1/_2\,\beta / \omega)\, e^{-1/2} = \gamma_0$. Every straight line $\gamma = \gamma_1 < \gamma_0$ intersects curve $\gamma = \varphi(r_2)$ at two points with abscissas $r_2' = r_2'(\gamma_1)$ and $r_2'' = r_2''(\gamma_1)$ ($r_2' < {}^1/_2 < r_2''$).

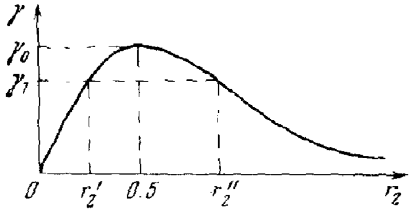

Fig. 3

We use the method of contractive mappings for formulating the following theorem.

Theorem 1. Solution $\nu^* = (\mu_0^*, \mu_1^*, \mu_2^*)$ of Eq. (2.15) exists in space $B_2' = C \times C \times [0, r_2']$ when $\gamma < \gamma_1 < \gamma_0$. In space $B_2'' = C \times C \times [0, r_2'']$ ($B_2' \subset B_2''$)

this solution is unique. It can be found by the method of successive approximations with the use of the following scheme

$$\nu^{(n)} = A\ (\nu^{(n-1)}) \qquad (n = 1,\ 2,(..)) \tag{3.9}$$

and any initial approximation $\nu^{(0)} \in B_2'$. Estimate of the nth order approximation error is given by formula

$$\|\nu^* - \nu^{(n)}\|_B \leqslant \frac{\alpha_1^n}{1-\alpha_1}\|\nu^{(0)} - A(\nu^{(0)})\|_B,\ \alpha_1 = \alpha(r_2^1), \quad \nu^{(0)} \in B_2^1 \tag{3.10}$$

4. If $\nu^* = (\mu_0^*, \mu_1^*, \mu_2^*)$ is a solution of Eq. (2.15), which belongs to space B, then it can be readily shown that function

$$f^*(\zeta) = K\left(P_0(\mu_0^*, \mu_2^*),\ P_1(\mu_1^*, \mu_2^*)\right) \tag{4.1}$$

is a solution of the boundary value problem (2.2) – (2.4). The converse statement that, if function $f^*(\zeta)$ solves the boundary value problem (2.2) – (2.4) and μ_0^*, μ_1^* and μ_2^* are determined by equalities (2.5) and (2.6), then $\nu^* = (\mu_0^*, \mu_1^*, \mu_2^*)$ is a solution of Eq. (2.15) and $\nu^* \in B$ is also valid. Hence the boundary value problem (2.2) – (2.4) has no solutions different from those derived by solving Eq. (2.15) by formula (4.1).

Let $\nu^{(n-1)} = (\mu_0^{(n-1)}, \mu_1^{(n-1)}, \mu_2^{(n-1)})$ $(n = 1,2, \ldots)$. We introduce the notation

$$f^{(n)}(\zeta) = K\left(P_0(\mu_0^{(n-1)}, \mu_2^{(n-1)}),\ P_1(\mu_1^{(n-1)}, \mu_2^{(n-1)})\right) \tag{4.2}$$
$$\lambda_k^{(n)} = \operatorname{Re} f^{(n)}(k\pi\tau/2 + \xi),\ \lambda_k^{\prime(n)} = d\lambda_k^{(n)}/d\xi \quad (k = 0,\ 1)$$

Taking into account (2.9) – (2.12) which define the properties of operator K, we obtain

$$\operatorname{Re} f^{(n)}(\pi/2) = 0,\ \lambda_k^{\prime(n)} = P_k(\mu_k^{(n-1)}, \mu_2^{(n-1)}) \quad (k = 0,1) \tag{4.3}$$
$$\operatorname{Im} f^{(n)}(\xi) = D_0(P_0(\mu_0^{(n-1)}, \mu_2^{(n-1)})) - D_1(P_1(\mu_1^{(n-1)}, \mu_2^{(n-1)}))$$
$$\operatorname{Im} f^{(n)}\left(\frac{\pi\tau}{2} + \xi\right) = D_1(P_0(\mu_0^{(n-1)}, \mu_2^{(n-1)})) - D_0(P_1(\mu_1^{(n-1)}, \mu_2^{(n-1)}))$$
$$3\operatorname{Re} f^{(n)}\left(\frac{\pi\tau}{2} + \frac{\pi}{2}\right) = D_2(P_1(\mu_1^{(n-1)}, \mu_2^{(n-1)})) - D_2(P_0(\mu_0^{(n-1)}, \mu_2^{(n-1)}))$$

From this with allowance for (3.9) and (2.14) we obtain

$$\operatorname{Im} f^{(n)}(k\pi\tau/2 + \xi) = \mu_k^{(n)} \qquad (k = 0,1)$$
$$3\operatorname{Re} f^{(n)}(\pi\tau/2 + \pi/2) = \mu_2^{(n)}$$

It is not difficult to verify that

$$\max \zeta \in \overline{D}\,|f^*(\zeta) - f^{(n)}(\zeta)| \leqslant \tag{4.4}$$
$$\max k = 0,\ 1\left(\|\lambda_k^* - \lambda_k^{(n)}\|_C + \|\mu_k^* - \mu_k^{(n)}\|_C\right)$$

where $k = 0,1$ and

$$\|\lambda_k^* - \lambda_k^{(n)}\|_C \leqslant \frac{k}{3}\|\mu_2^* - \mu_2^{(n)}\|_E + \frac{\pi}{2}\left\|\frac{d\lambda_k^*}{d\xi} - \lambda_k^{\prime(n)}\right\|_C \tag{4.5}$$

Using formulas (2.7), (4.3), (3.2), (3.3) and (3.7), from (4.4) and (4.5) for $\nu_0 \in B_2'$ and $\Upsilon < \Upsilon_1 < \Upsilon_0$ we obtain

$$\max \zeta \in \overline{D}\,|f^*(\zeta) - f^{(n)}(\zeta)| \leqslant \alpha_1\|\nu^* - \nu^{(n-1)}\|_B$$

which with allowance for (3.10) yields

$$\max \zeta \in \overline{D} \, | f^*(\zeta) - f^{(n)}(\zeta) | \leqslant \frac{\alpha_1^n}{1-\alpha_1} \| v^{(0)} - A(v^{(0)}) \|_B \qquad (4.6)$$

Let us denote by $M(r_2)$ that class of functions $f(\zeta)$ which are analytic in D, continuous in $\overline{D}$ and such that

$$3\operatorname{Re} f\left(\frac{\pi\tau}{2} + \frac{\pi}{2}\right) \leqslant r_2, \quad \frac{d}{d\xi} \operatorname{Re} f\left(\frac{k\pi\tau}{2} + \xi\right) \in C \quad (k = 0, 1)$$

On the basis of these results we can formulate the following fundamental theorem.

Theorem 2. Solution of the boundary value problem (2.2) – (2.4) for $\gamma < \gamma_1 < \gamma_0$ exists in the class $M(r_2')$, and in the class $M(r_2'')$ it is unique. The solution can be derived by the method of successive approximations with the use of formulas (4.2) and (3.9) for any $v^{(0)} \in B_2'$. Estimate of the nth approximation error is given by formula (4.6).

With $f^*(\zeta)$ known, any geometric and kinematic properties of the flow can be determined by formulas (1.1), (1.2), (1.4) and (1.6). The Bernoulli equation provides its dynamic properties.

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